SLAC–PUB–KCL-TH-00-65
hep-th/0012121
December 2000

Goldstone-Soliton Interactions and Brane World Neutrinos

N.D. Lambert and P.C. West*

Department of Mathematics
King's College, London
WC2R 2LS
England

ABSTRACT

We discuss the interactions of Goldstone particles with solitonic states. We observe that, contrary to the familiar situation in the vacuum sector, the Goldstone particles can have non-derivative interactions with the solitons. This result is applied to brane physics and in particular leads to the possibility that neutrinos in brane world scenarios are Goldstone particles for broken supersymmetry.

⋆ lambert,pwest@mth.kcl.ac.uk

1. Introduction

One of the earliest developments in supersymmetry was the paper by Volkov and Akulov [1] which found a non-linear realisation of the supersymmetry algebra that was reported in the early paper of Golfand and Likhtman [2]. This quantum field theory described a massless Fermion interacting with itself through derivative interactions. By analogy with the case for internal symmetries, the the Fermion described by this non-linear realisation was to be thought of as the Goldstone particle that should arise if supersymmetry is spontaneously broken. These authors then proposed that the Goldstone particle corresponding to the spontaneous breaking of supersymmetry was the neutrino. It was later shown [3] that supersymmetry did indeed obey a Goldstone theorem and spontaneously broken supersymmetry does inevitably lead to a massless particle of spin 1/2, the Goldstino. However, a problem arose with its interpretation as a neutrino. For the case of internal symmetries it was well-known that the Goldstone particles had only derivative interactions with themselves and all the other particles in the theory. It was shown [4] that this theorem also applied to the Goldstino but it was known from experiment that the neutrino had non-vanishing cross sections in the limit of zero momentum with the other leptons. Thus the hypothesis that the neutrino was a Goldstino was seen to be in contradiction with experiment and laid to rest.

This failure posed a problem for model building using spontaneously broken supersymmetry since this mechanism inevitably lead to a Goldstino and this must be a massless particle that had not been observed so far. Later, mainly motivated by the tight pattern of masses that occur when spontaneously breaking supersymmetry in theories of rigid supersymmetry, model builders turned to theories that involved supergravity. In this case, supersymmetry is locally realised, so that the corresponding Goldstino will be absorbed by the gravitino and the apparent naive contradiction with observation is removed.

Within the past two years a significant amount of research has concentrated on the idea that branes in a higher dimensional spacetime are relevant for phenomenology (for example see [5,6,7,8,12]). A main motivation for the recurrence of this idea is that in string theory Yang-Mills fields are naturally confined to the worldvolume of branes. In these scenarios gravity is still a bulk field in all of spacetime. In this way of viewing things, the world we observe may emerge from a fundamental theory in a very different way than from the standard compactification methods.

A specific model of this type that seems natural from the point of view of M-theory is to consider an M-fivebrane which is wrapped on a two-cycle in a compactification of M-theory leaving a four dimensional worldvolume that is identified with Minkowski space [9]. For example M-theory compactified on a manifold of G_2 holonomy has four-dimensional $N = 1$ supersymmetry. The supersymmetries of M-theory can be broken by the wrapping or self-intersection of the fivebrane and by the background spacetime. One appealing possibility is to break all the supersymmetries and to try to find a low energy theory which is just the standard model and no more [9].

In any brane world scenario it is natural to imagine that some supersymmetry is preserved by the bulk spacetime but is broken by the brane on which the standard model lives. By analogy with the Goldstone theorem one would expect to see massless Fermionic fields on the brane corresponding to the breaking of bulk supersymmetry by the brane. As already mentioned, normally in supergravity the spin 1/2 Goldstinos are eaten by the gravitinos in a super-Higg's mechanism. Such a mechanism seems implausible in the case of D-branes however since the spin 1/2 Goldstinos are confined to the brane whereas the gravitini propagate in the bulk spacetime.

Therefore an apparently robust feature of these brane world scenarios is the prediction of massless, uncharged Goldstone Fermions. In this way one is naturally returned to the idea of Volkov and Akulov but then also into its conflict with experiment.

We note that an interesting Higg's mechanism does exist for smooth domain walls whose transverse space is a circle [12]. In particular the lowest mode of the Kaluza-Klein vector field in the bulk eats the Goldstone boson corresponding to broken translational symmetry around the circle. It is natural to also expect that this mechanism holds for the case of supersymmetries, so that the would-be Goldstone Fermions are eaten by the lowest mode of the bulk gravitini. However the topological stability of such domain walls is questionable since continuity and perodicity of the scalars around the compact direction imply that the domain wall has no topological charge (see for example [13]). In any case, in this paper we hope to convince the reader that there is an alternative resolution to the Goldstino puzzle in which the appearance of massless, uncharged Fermions such as neutrinos is rather natural.

There is another example in which the purely derivative interactions of Goldstone modes needs to be questioned. We recall that an M-fivebrane wrapped on a non-compact Riemann Surface can be related by M-theory/Type IIA duality to $N = 2$ Yang-Mills gauge theory [10]. In particular the low energy dynamics of M-fivebrane precisely reproduce the Seiberg-Witten effective action for $N = 2$ Yang-Mills theory [11]. The massless Fermions which appear in the M-fivebrane effective action are Goldstinos for broken supersymmetry and these must be identified with the massless Fermions that appear in the $N = 2$ Yang-Mills theory after the gauge group is spontaneously broken. However, in the full $N = 2$ Yang-Mills Lagrangian these massless Fermions have non-derivative couplings to the charged states (e.g. the $W^{\pm}$). Therefore M-theory/Type IIA duality implies that these Goldstinos have non-derivative interactions with other states on the M-fivebrane. Indeed, even without assuming such a duality, it is certainly the case that the low energy effective action for the Goldstino modes of the M-fivebrane is identical to the effective action of a theory in which these Fermions have non-derivative interactions.

There is an important connection between these two cases. In the low energy equations of motion of an M-fivebrane (and D-branes too) only the field strength, and not the gauge field, appears. Thus none of the low energy states are charged. All charged states that appear in the low energy dynamics of branes must arise as soliton solutions [9]. Indeed another related question is how do these charged soliton states couple to the gauge potential. It is now clear that a way out of both of these dilemmas is to postulate that Goldstone modes can have non-derivative interactions with soliton states. Therefore in this paper we will explicitly exhibit this mechanism for a general theory. We then argue that the goldstino modes corresponding to the breaking of supersymmetry by the brane will indeed have non-derivative couplings to the charged states that also live on the brane.

The rest of this paper is organised as follows. In the next section we will briefly review Goldstone's theorem, quantisation about soliton solutions and apply these ideas to the case of branes. In section three we will consider the case of theories which simultaneously admit both soliton solutions and Goldstone modes, the best known example perhaps being the Skyrme model. In particular we will explicitly demonstrate a mechanism for non-vanishing Goldstone/soliton scattering at zero momentum (although this does not occur in the Skyrme model). In section four we will then focus the general discussion to the specific case of the M-fivebrane. Here we will discuss the resolution of the apparent contraction with type IIA string theory and in addition discuss the non-derivative interactions that worldvolume Goldstinos have with charged states in phenomenological brane models. We note here that in this paper we will give general arguments to establish the existence and origins of non-derivative interactions between Goldstone particles and solitons. However, we will not provide a detailed analysis for all cases, such as broken supersymmetries. We expect that analogous result will follow this and other cases by a straightforward extension of the discussion presented here.

2. Branes and Goldstone Particles

In this paper we will mainly work in D dimensions. We use $\mu, \nu = 0, 1, 2, , ..., D-1$. A D-vector will be denoted by k^μ or just k. We will also need to consider the purely spatial components of a vector which we denote by $\vec{k}$. We use the letters $i, j, k, ...$ to label the various fields that appear and $A, B, C...$ to label any global symmetries.

Although it is widely known, it will be helpful if we summarise an elementary proof of Goldstone's theorem in the case of an internal symmetry which is spontaneously broken. Suppose that we consider the effective action $\Gamma[\phi]$. Let us further suppose that the effective action is invariant under a symmetry $\delta\phi^i = \omega^A f^i_A(\phi)$ so that

$$\int d^D x \frac{\delta\Gamma}{\delta\phi^i} \omega^A f^i_A(\phi) = 0 \ . \tag{2.1}$$

If consider the case of zero momentum on all external legs then $\Gamma[\phi] = -V(\phi)$ is just the effective potential and is independent of x^μ. Therefore we can drop the integral from (2.1). Differentiating with respect to ϕ^j we find, at zero momentum,

$$\frac{\partial^2 V}{\partial\phi^i \partial\phi^j} \omega^A f^i_A(\phi) + \frac{\partial V}{\partial\phi^i} \omega^A \frac{\partial f^i_A}{\partial\phi^j} = 0 \ . \tag{2.2}$$

Upon setting $\phi^i = \phi^i_0$, where ϕ^i_0 is a constant field configuration that minimises the effective potential, the second term in (2.2) vanishes. Since the first term is just the mass matrix, we see that there is one massless particle for every broken symmetry, which is precisely Goldstone's theorem [14].

Differentiating (2.2) with respect to ϕ^i, multiplying by $\omega^B f^j_B(\phi) \omega^C f^k_C(\phi)$ and taking $\phi^i = \phi^i_0$ we find that

$$\frac{\partial^3 V(\phi_0)}{\partial\phi^i \partial\phi^j \partial\phi^k} \omega^A f^i_A(\phi_0) \omega^B f^j_B(\phi_0) \omega^C f^k_C(\phi_0) = 0 \ . \tag{2.3}$$

Thus we see that at zero momentum the Goldstone three-point function vanishes. Clearly we could keep differentiating and deduce that the Goldstone N-point function vanishes at zero momentum for all N.

In addition we could also repeatedly differentiate (2.1) and equation (2.2) with respect to other fields ψ that occur in the theory and we would conclude that their coupling to the Goldstone Bosons also vanishes at zero momentum.

We will be interested in the Goldstone Bosons that occur in the presence of branes. Branes can be thought of a solitonic solutions to the classical equations of motion and before proceeding it will be helpful to remind the reader of how to include solitons in the quantum theory. Denoting the solitonic solution by ϕ_s^i, we define the fluctuations about the soliton by

$$\phi^i = \phi_s^i + \phi_q^i \ , \tag{2.4}$$

where ϕ_q^i is the quantum field. Next we expand the effective action

$$\begin{aligned}\Gamma[\phi_s + \phi_q] = \Gamma[\phi_s] + \int d^D x_1 \frac{\delta\Gamma[\phi_s]}{\delta\phi^i(x_1)} \phi_q^i(x_1) \\ + \frac{1}{2}\int d^D x_1 d^D x_2 \frac{\delta^2\Gamma[\phi_s]}{\delta\phi^i(_1)\delta\phi^j(x_2)} \phi_q^i(x_1)\phi_q^j(x_2) + \dots \ .\end{aligned} \tag{2.5}$$

Here the ellipsis indicates the higher order terms in ϕ_q^i which we can ignore at lowest order in perturbation theory. The linear term in ϕ_q^i vanishes since the soliton is a solution of the equations of motion. To proceed further we write ϕ_q^i in terms of the complete set of solutions to the linearised equations in the presence of the soliton; that is the equations

$$\int d^D x_2 \frac{\delta^2\Gamma[\phi_s]}{\delta\phi^i(x)\delta\phi^j(x_2)} \eta^j(x_2) = 0 \ . \tag{2.6}$$

For a static soliton these equations become[⋆]

$$\int d^{(D-1)}\vec{x}_2 \frac{\delta^2\Gamma[\phi_s]}{\delta\phi^i(x)\delta\phi^j(x_2)} \eta^j(\vec{x}_2) = -E^2\eta^i \ , \tag{2.7}$$

where we have taken $\eta^i(x) = e^{-iEt}\eta^i(\vec{x})$. Labelling the solutions by the index I,

⋆ We have assumed for the sake of simplicity that the linearised equation can be written in the form $-\partial_0^2\phi_q^i + \dots$ where the ellipses denote terms that don't involve time derivatives.

i.e. η^i_I, we write

$$\phi^i_q(x) = \sum_I \eta^i_I(\vec{x}) a^i_I(t) \ . \tag{2.8}$$

The dynamics are now described by the variables $a^i_I(t)$ and their action is found from the original functional integral by substituting the expression for ϕ^i_q above into equation (2.4) and keeping terms second order in a^i_I.

If the soliton solution depends on only some of the spatial coordinates, as is generally the case for branes, then equation (2.6) is written differently. In particular, if the solitonic solution depends only on the coordinates $y^\mu,\ \mu = p+1, \ldots, D-1$ and we denote the remaining coordinates, including time, by $x^\mu,\ \mu = 0, 1, 2, \ldots, p$ then equation (2.6) takes the form

$$\int d^{(D-1-p)}\vec{x}_1 \frac{\delta^2 \Gamma[\phi_s]}{\delta\phi^i(x)\delta\phi^j(x_1)} \eta^j = k^2_{||} \eta^i \ , \tag{2.9}$$

where $k^2_{||} = -(k^0)^2 + (k^1)^2 + \ldots + (k^p)^2$. The fluctuations are then expressed as

$$\phi^i_q = \sum_I \eta^i_I(y) a^i_I(x) \ . \tag{2.10}$$

Finally, we can derive the analogue of Goldstone theorem for branes. Let us suppose that the theory has a symmetry $\delta\phi^i = \omega^A f^i_A(\phi)$, where ω^A are constant parameters. As a result the effective action Γ obeys the equation

$$\int d^D x_1 \frac{\delta\Gamma}{\delta\phi^j(x_1)} \omega^A f^j_A(\phi) = 0 \ . \tag{2.11}$$

Differentiating with respect to $\phi^i(x)$ and taking the fields to be evaluated at the soliton solution ϕ^i_s we deduce that

$$\int d^D x_1 \frac{\delta^2 \Gamma[\phi_s]}{\delta\phi^i(x)\delta\phi^j(x_1)} \omega^A f^j_A(\phi_s) = 0 \ , \tag{2.12}$$

where we have again dropped a linear term that vanishes on-shell. If the soliton only depends on the coordinates y^μ, as described above, then this equation becomes

(2.9) but with $k_{||}^2 = 0$ and $\eta^i = \omega^A f_A^i(\phi_s)$. In other words, any symmetries of the action which are broken by the soliton lead to solutions of the linearised field equation which are independent of the worldvolume coordinates of the branes. Hence when expressing the fluctuation ϕ_q^i in terms of the complete set we find a terms of the form $\omega^A(x) f_A^i(\phi_s(y))$ where $\omega^A(x)$ describes one massless particle corresponding for each of the symmetries broken by the soliton. Thus to every symmetry broken by the solitonic solution we find a massless particle propagating on the space where the soliton lives. In the case of branes this means propagating in the worldvolume of the brane. The action of these particular modes is deduced from the original action and takes the form

$$\int d^{p+1}x \partial_\mu \omega^A \partial^\mu \omega^B g_{AB} \ , \tag{2.13}$$

where g_{AB} can be interpreted as a metric on moduli space and is determined by carrying out the y-integration. In the quantum theory it is this metric that enters into the norm of the states. We recognise these Goldstone modes as the collective coordinates discussed in the literature (for a review see [15]).

An exception to Goldstone's theorem for solitons occurs if the metric g_{AB} turns out not to be finite as a result of the y-integration. To illustrate this point consider the standard non-linear sigma model based on the coset G/H. Such a theory is described by the group element $g \in G$ and is invariant under $g \to g_0 g h$ where $g_0 \in G$ is a rigid transformation and $h \in H$ is a local transformation. The action takes the form

$$\int d^D x \sum_i (\mathrm{Tr}(g^{-1} dg X^i))^2 \ , \tag{2.14}$$

where X^i are the coset generators. Let us suppose we have a static solitonic solution then we are interested in modes where g_0 depends on t i.e. $g_0(t)$. At infinity $g \to 1$ and these modes have the action

$$\int dt \int d^{D-1}\vec{x} \sum_i (\mathrm{Tr}(g_0^{-1}(t) \dot{g}_0(t) X^i))^2 \ , \tag{2.15}$$

in this region. Clearly, this diverges for those elements g_0 that belong to the coset.

Of course such a simple sigma model action does not admit solitonic solutions above two dimensions. Indeed while Golstone modes require a spontaneously broken continuous symmetry with a corresponding continuous family of vacua, solitons are often associated with discrete vacua. Therefore it might seem contradictory to discuss both broken symmetry and solitons in the same theory. However such theories do exist. For example, one can add higher order terms to the non-linear sigma model action (2.14) which enable the existence of solitons. A familiar example of this type is provided by the Skyrme model in four dimensions. In these situations the quantum states are not normalisable and the corresponding modes must be dealt with differently.

Let us now briefly review the Skyrme model [16] in order to illustrate this point in detail and more importantly because, as we will see, this model is very analogous to the situation that occurs for branes. The action for the Skyrme model is given by

$$S_{Skyrme}[U] = \int d^4x \mathrm{Tr}\left(\frac{f_\pi^2}{16}\partial_\mu U \partial^\mu U^\dagger + \frac{1}{32e^2}[(\partial_\mu U)U^\dagger, (\partial_\nu U)U^\dagger]^2\right) \ , \tag{2.16}$$

where f_π and e are constants and $U(x)$ is an element of $SU(2)$. This action is clearly invariant under the group $SU_L(2) \times SU_R(2)$ which acts as $U \rightarrow A_L U A_R^\dagger$ where $A_{L/R} \in SU_{L/R}(2)$. However we must choose a vacuum configuration since any constant U has zero energy and can be used to define a vacuum state. Without loss of generality we choose the vacuum to be $U = 1$. Such a choice breaks the $SU_L(2) \times SU_R(2)$ symmetry to $SU_D(2)$ generated by taking $A_L = A_R$. Thus there will be three Goldstone Bosons corresponding to the broken symmetry. To exhibit these modes we write

$$U(x) = e^{i\pi^i(x)T^i} \ , \tag{2.17}$$

where T^i, $i = 1, 2, 3$ are the Pauli matrices and π^i are the Goldstone modes, i.e. the pions. It is clear from the action that the pions are massless and, in the limit of zero momentum, there are no interactions of the Goldstone modes among themselves.

As pointed out by Skyrme many years ago the inclusion of the higher derivative term in (2.16) allows for static soliton solutions of the form

$$\pi^i = G(r)\frac{x^i}{r} \ , \tag{2.18}$$

where $r = |\vec{x}|$ and we impose the boundary condition that $G(0) = \pi$ and $G(r) \rightarrow 0$ as $r \rightarrow \infty$. The solutions to these equations have a finite energy, are topologically stable and well-known as "Skyrmions".

It is instructive to examine the Skyrmion zero modes. First there are the zero modes associated with the breaking of translation invariance, which occur for any finite energy soliton and are given by $f^i_\mu(\phi_s) = \partial_\mu \phi^i_s$. In addition if U_s is a Skyrmion then clearly $A_L U_s A_R^\dagger$ also solves the field equations for any constant choices of A_L and A_R. However let us promote A_L and A_R to time-dependent matrices. Substituting $A_L(t) U_s A_R^\dagger(t)$ into the action (2.16) we may determine an effective action for the modes A_L and A_R. This leads to a rather complicated expression. However we can see that not all such modes have a finite kinetic energy. In particular the leading order behaviour for large r is

$$\begin{aligned} S_{eff} &= S_{Skyrme}[A_L(t) U_s A_R^\dagger(t)] \\ &= \frac{\pi f_\pi^2}{4} \int dt \int r^2 dr \mathrm{Tr}\left(\left[\dot{A}_L \dot{A}_L^\dagger + \dot{A}_R^\dagger \dot{A}_R - \dot{A}_L^\dagger A_L A_R^\dagger \dot{A}_R - \dot{A}_L \dot{A}_R^\dagger A_R A_L^\dagger \right] \right) \\ &\quad + \dots \ , \end{aligned} \tag{2.19}$$

where a dot denotes a time derivative and the ellipsis refers to terms with smaller powers of r in the limit $r \to \infty$. Thus we see that unless $A_L = A_R$ the spatial integral will diverge. Indeed the dominant terms at large r are simply those of a non-linear realisation as discuss above. Furthermore one can check that if $A_L = A_R$ then the sub-leading terms in (2.19) are finite [17]. Therefore only the zero-modes corresponding to $SU_D(2)$ can be associated with the motion of the soliton. The broken symmetries generate new solutions to the field equations which cannot be simply interpreted as dynamical states of the Skyrmion.

This is clearly a general phenomenon. If the field equations of a theory are invariant under a continuous symmetry group then any soliton will come with zero modes: i.e. by acting on a soliton with the symmetry generators we will obtain a new solution to the field equations (unless the soliton itself is invariant under the symmetry in question). However it is important to distinguish between what might be called genuine zero-modes of a soliton and non-genuine zero-modes. Genuine zero-modes have a finite kinetic term. Thus these modes appear in the low energy motion of the soliton in the form of collective coordinates. However there may also be zero-modes which do not have a finite kinetic term and so cannot be interpreted as collective coordinates. We call these non-genuine zero-modes.

If there are non-genuine zero modes then we will find solutions to the field equations which cannot be associated with the motion of the soliton. Acting on the vacuum with the broken symmetry generators produces a Goldstone state.

Therefore we intuitively expect that acting on a soliton with a broken symmetry generator (i.e. non-genuine zero-mode) produces a soliton/Goldstone state. We will show in the next section that this is in fact just a soliton/Goldstone scattering state at zero momentum.

3. Goldstone/Soliton Interactions at Zero Momentum

In this section we will now consider the interactions of Goldstone modes with solitons in a general quantum theory. We will be particularly interested in the scattering of Goldstone modes off a soliton at zero momentum. In the beginning of section two we explained why Goldstone particles had only derivative interaction with themselves and all other particles. This argument was based on the effective action viewpoint. Since the soliton is not represented by its own field, that derivation does not apply to the scattering of Goldstone particles off solitons. In fact we will see that the result does not hold in general and in some cases one indeed finds non-derivative Goldstone soliton interactions.

We take the approach to solitons reviewed in [15]. We imagine that the theory has a symmetry which is spontaneously broken leading to a corresponding Goldstone particle. These modes are included in the generic symbol ϕ^i. We suppose that the theory also possess a stable classical soliton solution which we denote by ϕ_s^i. Corresponding to this classical solution there is an associated quantum state of momentum $\vec{p}$ denoted by $|\vec{p}>_s$. We normalise the Lagrangian so that the fields occur in the combination $g\phi^i$, where g plays the role of a coupling constant and we include an overall factor of g^{-2} in front of the action

$$S = \frac{1}{g^2}\int d^D x \mathcal{L}(g\phi^i) \ . \tag{3.1}$$

Solutions to the equations of motion, in particular the solitons ϕ_s^i, are then of order g^{-1} and the mass of the soliton states are of order g^{-2}.

We must also consider states which contain the soliton and the elementary particles that occur in the sector of the theory that has no solitons. The latter include the Goldstone particles of the theory. The simplest examples are the states $|\vec{p}; \vec{k}_1, \vec{k}_2, ... >_s$ where $\vec{p}$ is the momentum of the soliton and $\vec{k}_1, \vec{k}_2, ...$ are the momenta of the Goldstone particles or other fundamental particles. The connected components of the matrix elements between two states are assumed to be of order g^{n-1} where n is the number of elementary particles involved. We also assume that the soliton is stable so that the matrix element between a state in the soliton section and a state in the vacuum sector vanishes. With these assumptions one can show that, to lowest order in g,

$$\phi^i(x) = \frac{1}{(2\pi)^{D-1}} \int d^{D-1}(p-q) e^{i(\vec{p}-\vec{q})\cdot\vec{x}} {}_s < \vec{p}|\phi^i(0)|\vec{q} >_s , \tag{3.2}$$

solves the field equations and we therefore identify it with the soliton solution $\phi^i_s(x)$. Furthermore, if we introduce the Fourier transform of the matrix element with a Goldstone state

$$\eta^i_k(\vec{x}) = \frac{1}{(2\pi)^{D-1}} \int d^{D-1}(p-q) e^{i(\vec{p}-\vec{q})\cdot\vec{x}} {}_s < \vec{p}|\phi^i(0)|\vec{q};\vec{k} >_s , \tag{3.3}$$

then one can show that $e^{-iE_k t}\eta^i_k(\vec{x})$ solves the linearised equation of motion (2.6) in the background of the soliton.

We now will derive the interaction of the soliton with the Goldstone particles at zero momentum. We begin with the relation

$$[Q_A, \phi^i] = i\delta_A \phi^i , \tag{3.4}$$

where Q_A is one of the symmetry generators of the theory that is spontaneously broken. Taking the scalar product of the left hand side with solitonic states and writing the charge as the integral of its current j^μ_A we find that

$$\int d^{D-1}\vec{x} \ {}_s < \vec{p}|j^0_A(\vec{x},t)\phi^i|\vec{q} >_s - {}_s < \vec{p}|\phi^i j^0_A(\vec{x},t)|\vec{q} >= i \ {}_s < \vec{p}|\delta_A\phi^i|\vec{q} >_s . \tag{3.5}$$

Using translational invariance we choose to evaluate ϕ^i at the origin of the coordinates, i.e. $\phi^i = \phi^i(0)$. Sandwiching with a complete set of states $|n>$, the left-hand side of (3.5) becomes

$$\sum_n \int d^{D-1}\vec{x} \ {}_s < \vec{p}|j^0_A(\vec{x},t)|n >< n|\phi^i|\vec{q} >_s - {}_s < \vec{p}|\phi^i|n >< n|j^0_A(\vec{x},t)|\vec{q} >_s \tag{3.6}$$

In the complete set, the only states that contribute are the one soliton states. Goldstone's theorem asserts that acting on the vacuum with a broken j^0_A creates a massless particle which carries the same quantum numbers as the current. In the soliton sector of the theory we can use cluster decomposition to argue that a broken j_A^0 also creates a Goldstone mode in the soliton background, since far from the soliton core it must act in the same way as it does in the vacuum. Consequently, for a broken symmetry Q_A, the intermediate matrix element ${}_s<\vec{p}|j_A^0(\vec{x},t)|n>$ will be non-vanishing only if the states $|n>$ are a one soliton state with an appropriate Goldstone particle, that is the state denoted $|\vec{p'};\vec{k'}>_s$. In some cases, for example if all the solitons carry the same quantum numbers, this may also be seen as a consequence of the conservation of the quantum number associated to j_A^μ. Using translational symmetry, that is

$$ {}_s<\vec{p}|A(\vec{x},t)|n>={}_s<\vec{p}|A(0,0)|n>e^{-i(E_p-E_{p_n})t}e^{i(\vec{p}-\vec{p}_n)\cdot\vec{x}} \ , \tag{3.7} $$

we may insert the Goldstone/soliton states and carry out the $\vec{x}$ integration. We then find that the left hand side of equation (3.5) becomes

$$ \begin{aligned} &(2\pi)^{D-1}\int\frac{d^{D-1}\vec{p'}}{2E_{p'}}\int\frac{d^{D-1}\vec{k'}}{2E_{k'}}\Big\{ \\ &\quad\delta^{D-1}(\vec{p}-\vec{p'}-\vec{k'})e^{-i(E_p-E_{p'}-E_{k'})t}{}_s<\vec{p}|j_A^0(0,0)|\vec{p'};\vec{k'}>_s{}_s<\vec{p'};\vec{k'}|\phi^i|\vec{q}>_s \\ &\quad-\delta^{D-1}(\vec{p'}-\vec{k'}-\vec{q})e^{i(E_q-E_{p'}-E_{k'})t}{}_s<\vec{p}|\phi^i|\vec{p'};\vec{k'}>_s{}_s<\vec{p'};\vec{k'}|j_A^0(0,0)|\vec{q}>_s\Big\} \ . \end{aligned} \tag{3.8} $$

We will evaluate the above quantity to lowest order in the coupling g. At this order the matrix element ${}_s<\vec{p}|j_A^0(0,0)|\vec{p'};\vec{k'}>_s$ takes the form of a disconnected diagram, that is

$$ {}_s<\vec{p}|j^0(0,0)_A|\vec{p'};\vec{k'}>_s={}_s<\vec{p}|\vec{p'}>_s<0|j_A^0(0,0)|\vec{k'}> \ . \tag{3.9} $$

This quantity is of order g^0. Let us assume that the current j_A^μ carries no Lorentz index other than μ. Imposing Lorentz invariance we deduce that $<0|j_A^\mu(0)|\vec{k'}>=iF_Ak'^\mu$ where F_A is constant of order g^0 which we may take to be real. Hence we obtain

$$ {}_s<\vec{p}|j_A^\mu(0,0)|\vec{p'};\vec{k'}>_s=2iF_Ak'^\mu E_p\delta^{D-1}(\vec{p}-\vec{p'}) \ . \tag{3.10} $$

Substituting in this expression, and using the on-shell relation $E_{k'}=|\vec{k'}|$ for massless fields, we arrive at

$$i(2\pi)^{D-1} F_A \lim_{k' \to 0} \Big({}_s < \vec{p}; \vec{k'} | \phi^i | \vec{q} >_s + {}_s < \vec{p} | \phi^i | \vec{q}; \vec{k'} >_s \Big) = i \ {}_s < \vec{p} | \delta_A \phi^i | \vec{q} >_s \ . \tag{3.11}$$

We recall that the generator Q_A is one of the symmetry generators that is spontaneously broken in the vacuum sector and as a consequence leads to Goldstone particles. We have assumed that the soliton also breaks this symmetry, i.e. $Q_A | \vec{p} >_s \neq 0$, and as a result ${}_s < \vec{p} | \delta_A \phi^i | \vec{q} >_s$ is non-vanishing. In fact, to lowest order, ${}_s < \vec{p} | \delta_A \phi^i | \vec{q} >_s$, or more precisely its Fourier transform, is a solution to the linearised equation of motion about the soliton background. Hence we conclude that the matrix element ${}_s < \vec{p}, \vec{k'} | \phi^i | \vec{q} >_s$ at zero Goldstone momentum is related to the variation of the soliton under a broken symmetry.

The above argument parallels the classic proof of Golstone's theorem [14]. In this case one begins with the same relationship of equation (3.5) but takes the matrix element to be between the vacuum states rather than the solitonic states. One then proves that there exist massless particles with the same quantum numbers as the broken generators. This follows as one shows that there must exist a massless particle $| \vec{k} >$ in the complete set such that $< 0 | j^0 | \vec{k} >$ and $< \vec{k} | \phi^i | 0 >$ are non-vanishing.

In fact by analogy with the original proof of Goldstone's theorem we can prove a slightly stronger result by considering the time derivative of the relation $[Q, \phi^i] = \delta \phi^i$. Writing $\partial_0 Q$ as an integral over $\partial_0 j^0$ we can express the volume integral as a surface integral at infinity. Using the fact that operators at space-like separation commute, we conclude that ${}_s < \vec{p} | \delta \phi^i | \vec{q} >_s$ is independent of time. On the other hand if we evaluate (3.8) then the requirement of time independence implies the massless on-shell condition $\lim_{\vec{k} \to 0} E_{\vec{k}} = 0$. Therefore we learn that the intermediate particles must be massless modes.

For completeness we consider the analogous proof if we choose a symmetry Q_A that is unbroken in the vacuum. In this case it is clear that the internal symmetry will lead to a moduli space of solitons. In the quantum theory there will be discrete orthonormal states, represented by wave functions on this moduli space, which are interpreted as distinct soliton states. Although all these soliton states carry the same soliton number, they may carry different quantum numbers such as spin and, in the case of the Skyrme model, isopin [17]. Therefore we introduce the indices $\alpha, \beta, ...$ and label the soliton states as $| \vec{p}, \alpha >$. Note that this does not substantially affect the previous discussion since, upon using (3.9), we would learn that F_A is replaced by matrix $F_A^{\alpha\beta}$ which is diagonal. In which case the previous discussion applies for each type of soliton separately.

Returning to the argument we now see that there is no Goldstone particle in the intermediate state, at least not at lowest order in g, since there is no corresponding Goldstone mode in the vacuum sector. Rather we would find the intermediate state is another soliton $|\vec{p'},\gamma>_s$. Assuming that the charge Q_A and solitons are scalars, Lorentz invariance now restricts the soliton/current/soliton matrix element to be of the form

$$ {}_s<\vec{p},\alpha|j_A^\mu(0,0)|\vec{p'},\beta>_s = G_A^{\alpha\beta}(p+p')^\mu\delta^{D-1}(\vec{p}-\vec{p'}) \ , \tag{3.12} $$

with $G_A^{\alpha\beta}$ a constant Hermitian matrix of order g^0. Note that Lorentz invariance alone also allows for a term proportional to $(\vec{p}-\vec{p'})^\mu$ but this term vanishes if j_A^μ is conserved. In addition elements of $G_A^{\alpha\beta}$ where the solitons labelled by α and β have the different masses must also vanish. If we now continue with the argument as above we conclude that

$$ \begin{aligned} (2\pi)^{D-1}\sum_\gamma \Big({}_s<\vec{p},\alpha|\phi^i|\vec{q},\gamma>_s G_A^{\gamma\beta} & -G_A^{\alpha\gamma}\, {}_s<\vec{p},\gamma|\phi^i|\vec{q},\beta>_s\Big) \\ & = i\ {}_s<\vec{p},\alpha|\delta_A\phi^i|\vec{q},\beta>_s \quad . \end{aligned} \tag{3.13} $$

In other words we simply find that the unbroken symmetries are linearly realised on the solitonic states. In terms of matrices we see that the variation of ${}_s<\vec{p},\alpha|\delta_A\phi^i|\vec{q},\beta>_s$ is given by its commutator with $G_A^{\alpha\beta}$.

Our last step is to determine the scatting amplitudes from the matrix elements ${}_s<\vec{p}|\phi^i(l)|\vec{q}>_s$ and ${}_s<\vec{p}|\phi^i(l)|\vec{q};k>_s$ where l is the Goldstone momentum. For simplicity we will drop any reference to the soliton indices $\alpha,\beta,...$ since it will be clear that they will not affect the main point of the discussion. As is the case in standard scattering theory, if we view the states $|\vec{p}>_s$, $|\vec{p};\vec{k'}>_s$ and ${}_s<\vec{p}|$, ${}_s<\vec{p};\vec{k}|$ as incoming and outgoing respectively, then we may use the LSZ reduction formula to relate these elements to soliton/soliton/Goldstone and soliton/soliton/Goldstone/Goldstone scattering. These matrix elements can be determined from the matrix elements dicussed above by a Fourier transform and use of the formula (3.7)

$$ \begin{aligned} {}_s<\vec{p}|\phi^i(l)|\vec{q}>_s &= \int d^4x e^{-il\cdot x}e^{i(p-q)\cdot x}{}_s<\vec{p}|\phi^i(0)|\vec{q}>_s \\ {}_s<\vec{p}|\phi^i(l)|\vec{q};\vec{k}>_s &= \int d^4x e^{-il\cdot x}e^{i(p-q-k)\cdot x}{}_s<\vec{p}|\phi^i(0)|\vec{q};\vec{k}>_s \quad . \end{aligned} \tag{3.14} $$

In turn we may use (3.2) and (3.3) to express the right hand side of (3.14) in terms of the Fourier transforms of the soliton solution and the solution to the linearised equation respectively. The first of these equations simply involves the soliton solution ϕ_s^i whereas the second equation involves solutions to the linearised field equation in the background of the soliton. However here we are interested in the scattering at zero Goldstone momentum and from (3.11) we learn that the corresponding matrix elements are obtained by acting with the symmetry variation corresponding to the broken symmetry on the soliton solution. We therefore find, in the zero Goldstone momentum limit,

$$ {}_s < \vec{p}|\phi^i(l)|\vec{q} >_s = (2\pi)^D \delta(l^0)\delta^{D-1}(\vec{p}-\vec{q}-\vec{l}) \int d^{D-1}y e^{i(\vec{p}-\vec{q})\cdot\vec{y}} \phi_s^i(\vec{y}) \ , \tag{3.15} $$

and

$$ \begin{aligned} \lim_{k\to 0} \Big({}_s < \vec{p}|\phi^i(l)|\vec{q};\vec{k} >_s + {}_s < \vec{p};\vec{k}|\phi^i(l)|\vec{q} >_s \Big) = \\ \frac{2\pi}{F_A}\delta(l^0)\delta^{D-1}(\vec{p}-\vec{q}-\vec{l}) \int d^{D-1}y e^{i(\vec{p}-\vec{q})\cdot\vec{y}} \delta_A \phi_s^i(\vec{y}) \ , \end{aligned} \tag{3.16} $$

where δ_A is a broken symmetry variation. In deriving (3.15) we have used the fact that $E_{\vec{p}} - E_{\vec{q}} = 0 + \mathcal{O}(g^2)$ for any two solitons with the same rest mass.

The LSZ formula asserts that the disconnected part of the scattering amplitude is simply the residue of the $1/l^2$ term in the matrix element, up to a normalisation constant. Imposing the delta function constraint $l^0 = 0$, the three-point scattering amplitude between a Goldstone mode and two solitons comes from the $1/r^{D-3}$ term ($\ln r$ term in $D = 3$) in $\phi_s^i(r)$ where $r = |\vec{y}|$. Similarly the scattering amplitude of two Goldstone particles and two solitons, at zero Goldstone momentum, is given by the coefficient of the $1/r^{D-3}$ term ($\ln r$ term in $D = 3$) in the solution to the linearised equation of motion (2.6)

$$ \delta_A \phi_s^i = f_A^i(\phi_s) \ , \tag{3.17} $$

generated by a broken symmetry.

Let us return to our example of the Skyrme model in four dimensions. Here one finds that, under an infinitesimal $SU_L(2) \times SU_R(2)$ symmetry, the field transforms as

$$\delta\pi^i = (g_L^i - g_R^i)|\pi|\mathrm{cot}|\pi| - \pi^i \frac{(g_L^j - g_R^j)\pi^j}{|\pi|^2}(|\pi|\mathrm{cot}|\pi| - 1) - \epsilon^{ijk}(g_L^j + g_R^j)\pi^k \ , \quad (3.18)$$

where $A_{L/R} = e^{ig^i_{L/R}T^i}$ and $|\pi|^2 = \pi^i\pi^i$. The broken generators are $g_L^i = -g_R^i$ and we have seen that these give rise to non-genuine zero-modes. Substituting the Skyrmion soliton (2.18) into (3.16) and taking $g_L^i = -g_R^i$ we obtain

$$\delta\pi^i = 2g_L^j \left(\delta^i_j G\mathrm{cot}G - \frac{x^i x^j}{r^2}(G\mathrm{cot}G - 1)\right) \ , \quad (3.19)$$

which, as we have argued above, solves the linearised field equation (2.6). However a Skyrmion behaves like $\pi^i_s \sim G(r) = \mathcal{O}(1/r^2)$ as $r \to \infty$. Thus the scattering solutions (3.19) behave like $\delta\pi^i_s = 2g_L^j + \mathcal{O}(1/r^2)$ at infinity. Hence we conclude that in this model there are vanishing three-point and four-point scattering amplitudes between the Goldstone modes and the Skyrmion at zero momentum. Indeed this is the case [18].

On the other hand, for any soliton that carries electric or magnetic charge, the gauge field must have a non-vanishing $1/r^{D-3}$ term at infinity as a consequence of Gauss' law. We therefore see that these states have non-derivative soliton/soliton/gauge field couplings. Indeed this is just the familiar minimal coupling of a field to an electromagnetic potential. In the case of branes the low energy $U(1)$ gauge field arises as a Goldstone mode. In this way we see how the charged solitons can couple minimally to the gauge field. In the next section we will discuss in more detail other examples of non-derivative Goldstone/soliton scattering in the effective theory of branes.

4. Applications to Branes

In this section we wish to apply the general theory that we discussed above to the specific case of a wrapped M-fivebrane. Let us therefore begin with a discussion of M-fivebrane dynamics and in particular its relation to quantum gauge theory.

4.1 Review of the M-fivebrane and Gauge Dynamics

For obvious reasons we are most interested in M-fivebranes that have only four macroscopic spacetime dimensions. There are essentially two ways to realise this. The first is to consider intersecting M-fivebranes. In this case the four-dimensional spacetime is the intersection where the worldvolume fields become localised. Curiously, at least within the low energy approximation to the M-fivebrane, a smooth intersection is equivalent to a single M-fivebrane whose worldvolume appears wrapped on a calibrated (and in general non-compact) surface Σ [10,19,20,21,22]. Note that in this case the bulk spacetime need not have any non-trivial topology of its own, one is merely choosing a non-trivial embedding the M-fivebrane into the bulk. On the worldvolume of the M-fivebrane this intersection appears as a solitonic solution with only scalars active. The other possibility is simply to take the bulk eleven-dimensional spacetime to be of the form $\mathbf{R^4}\times\mathcal{M}$, where $\mathcal{M}$ is a non-trivial seven-manifold which has a topologically non-trivial two-cycle Σ over which we may wrap the M-fivebrane. Therefore both these constructions are similar in that they involve M-fivebranes which are in some sense "wrapped" over a two-dimensional surface Σ. However there is at least one crucial difference. In the first case, where the intersection is realised as a non-trivial embedding, there will generically be moduli of this embedding which will show up as massless fields on the M-fivebrane. On the worldvolume of the M-fivebrane the intersection appears as a soliton solution and these scalars may be thought of as Goldstone fields for broken translations. In the second case the M-fivebrane will wrap the two-cycle in such a way as to minimise its volume. This will be fixed by the properties of the bulk spacetime and therefore we don't expect any massless scalar moduli on the M-fivebrane.

The simplest example is the case of two static M-fivebranes intersecting over a common three-dimensional space, preserving eight supersymmetries. The M-fivebrane worldvolume description of this configuration is that of just a single M-fivebrane with two of its worldvolume dimensions wrapped over a non-compact Riemann surface [10,11]. The effective action for the zero-modes of the corresponding soliton can be constructed as outlined in section two and agrees precisely with the Seiberg-Witten low energy effective theory for $N = 2$ Yang-Mills theory [11].

The appearance of the Seiberg-Witten effective action is not a coincidence but rather a prediction of the duality between M-theory and type IIA string theory [10]. By compactifying on a circle the intersecting M-fivebranes are interpreted as intersecting D-fourbranes and NS-fivebranes in type IIA string theory. The description of D-branes in terms of open strings can be used to show that the brane dynamics are given by $N = 2$ Yang-Mills theory [10]. Therefore one expects that the low energy dynamics of the M-fivebrane should precisely reproduce the quantum low energy effective action of $N = 2$ Yang-Mills gauge theory. This example illustrates a deep relationship between the low energy dynamics of the M-fivebrane and quantum gauge field theories. It is also natural to wonder to what extent this relationship can be applied to more realistic gauge theories.

One of the central problems in string or M-theory is how to relate its effects to the observable world, in other words the standard model, in a convincing way. It could be that one has to understand much more about M-theory than we currently do in order to achieve this. However, it is also possible that we are at least in a position to find some convincing signs. There is a very large literature on relating supersymmetric theories, then string theories and more recently theories including branes to the world we observe. Almost all of these approaches have tried to construct not the standard model directly, but either the minimal supersymmetric extension of the standard model or some supersymmetric grand unified extension. As such these papers, including much of the recent work, is based on trying to find a realistic model which possesses hidden sectors or soft breaking terms.

In a previous paper [9] we discussed a more direct approach. The standard model involves the gauge groups $SU(2) \times U(1)$ broken to a $U(1)$ and a confined $SU(3)$. It also has chiral Fermions and no supersymmetry. These rather generic features arise in a natural way from intersecting branes. Hence the paper [9] addressed the question as to whether or not one could, in principle, find a wrapping of the M-fivebrane that lead directly to the standard model. That is to say to find a wrapping of the M-fivebrane whose effective theory is the same as the effective theory of the standard model.

One immediate problem which one faces is that the states that arise on D-branes are only charged with respect to two gauge groups, one for each end point of an open string. However quarks carry charges under all of the three simple factors of $SU(3) \times SU(2) \times U(1)$. In fact D-branes give rise to $U(N)$ gauge groups and the over-all $U(1)$ generally decouples. Therefore, as a first step in this direct approach, in [9] M-fivebrane configurations with gauge group $SU(3)\times SU(2)\times U(1)$ and containing states charged under all three groups were found. Furthermore the assignments of this "hypercharge" were derived from the brane physics and lead to a realistic spectrum. Recently the authors of [23,24] have taken other "bottom-up" approaches and [24] discussed mechanisms for hypercharge in perturbative type IIB string theory with similar features. It is possible that these mechanisms are related by T-duality.

More specifically, in [9] it was shown that M-fivebrane configurations exist with $N=2$ supersymmetry and the gauge group $SU(N_1)\times SU(N_2)\times ...\times SU(N_k)\times U(1)$. Furthermore these models possess hypermultiplet states in the $(\mathbf{N}_a,\mathbf{N}_b)$ of $SU(N_a)\times SU(N_b)$ for each pair of simple factors labelled by $a,b=1,2,...,k$. The $U(1)$ charge of these multiplets is determined to be $\pm(N_a^{-1}-N_b^{-1})$. In this way a natural toy model can be constructed with the gauge group $SU(3)\times SU(2)\times U(1)$ and "quark" multiplets in the representation $(\mathbf{3},\mathbf{2},\pm\frac{1}{6})$.

It is also possible to add n D-sixbranes into these models along the lines of [10]. Each D-sixbrane introduces k hypermultiplets in the $\mathbf{N}_a$ of $SU(N_a)$ which carry a $U(1)$ charge $\pm N_a^{-1}$. These multiplets are generically massive, however it is possible to tune some of them to be massless. Including a single D-sixbrane into our toy model we may obtain a massless "lepton" multiplet in the $(\mathbf{2},\pm\frac{1}{2})$ of $SU(2)\times U(1)^\star$ although it must also come with a massive mutliplet in the $(\mathbf{3},\pm\frac{1}{3})$ of $SU(3)\times U(1)$.

Of course such a toy model is still far from realistic. For instance it is non-chiral, has $N=2$ supersymmetry and it is not clear how three generations can be incorporated. Nevertheless we find it encouraging that a similar structure to what is found in the standard model arises so readily from branes. In [9] various steps were discussed that would lead to a more realistic model. For example the supersymmetry can be broken and the massless scalar modes removed by wrapping the M-fivebrane over a non-supersymmetric two-cycle in $\mathcal{M}$.

$\star$ We thank A. Uranga for pointing this out to us.

4.2 Goldstone Neutrinos

An M-fivebrane in eleven dimensions can be thought of as a solitonic solution of eleven-dimensional supergravity, or at a more fundamental level, of the underlying M-theory. It breaks translational invariance and half of the supersymmetries of M-theory. As expected we find that the dynamics of the fivebrane includes five Goldstone Bosons and sixteen Goldstone Fermions. There is also a self-dual three-form tensor field which arises from the breaking of certain automorphism symmetries of the M-theory algebra [25] and is the field strength for an Abelian two-form gauge field. In other words, all the massless modes of a brane in M-theory arise via non-linear realisations and therefore only have derivative couplings [25].

For wrapped M-fivebranes the resulting massless fields in the low energy effective action can therefore be interpreted as Goldstone particles which are confined to the four-dimensional worldvolume, in accordance with section two. In particular we expect Goldstone Fermions from the breaking of the worldvolume supersymmetries. Furthermore these Goldstinos cannot be absorbed by the gravitinos associated with the background spacetime as they are confined to the worldvolume of the self intersection. Put another way, they are not absorbed from the M-fivebrane viewpoint as this theory does not involve a dynamical gravitino. In addition there will generically be four-dimensional Abelian vector modes which arise from the two-form gauge field. All these degrees of freedom can be described by an effective action that, at least in principle, can be deduced from the dynamics of the M-fivebrane.

Note that, since the two-form arises as a Goldstone field on the M-fivebrane, it only has derivative interactions with itself and the other fields. That is to say it only appears in the equations of motion through its three-form field strength. Therefore any vector gauge fields that appear on the four-dimensional worldvolume do so through their field strengths. Hence all the fields that occur on the M-fivebrane are neutral under these $U(1)$'s. It follows that any charged states which arise in the dynamics of the M-fivebrane worldvolume must arise a solitons. One can think of these solitons as M-twobranes, whose boundaries are wrapped over one-cycles in Σ [10]. In the dual type IIA limit, obtained by compactification to ten dimensions, these vectors arise from the open strings ending on D-fourbranes. Some of these solitons have been studied in [26,27,28,29]. Their charge arises from the fact that M-twobranes couple directly to the self-dual gauge field of the M-fivebrane [30,31]. In particular for the $N = 2$ configurations discussed above, the states which correspond to the $W^{\pm}$ and monopoles can be found as soliton solutions on the M-fivebrane worldvolume [27,28,29].

This picture of charged states appearing as solitons is analogous to the situation with branes and Ramond-Ramond charges that emerged after the U-duality conjecture [32]. U-duality necessarily implies that there are states in string theory which are charged with respect to the Ramond-Ramond gauge fields. However it is well-known that no state in perturbative string theory can carry Ramond-Ramond charge, since these fields appear only through their field strengths. From the point of view of the low energy supergravity theory however one can readily find such states in the form of p-brane solitons [33]. It is only through the inclusion of D-branes [34] that such charged states are identifiable in the fundamental theory.

In the models discussed in [9] the Goldstone particles resulting from the wrapping of the M-fivebrane include Goldstone Fermions from the breaking of supersymmetry and vector particles corresponding to the unbroken $U(1)$'s. Since none of the massless fields on an M-fivebrane can carry the charge of the gauge fields, these Goldstone Fermions are neutral and could potentially be identified with neutrinos. Furthermore, as we have discussed above, in such a scenario any states which are charged under the low energy $U(1)$, such as the electron, can only arise as solitons of the M-fivebrane equations of motion.

It was noted [9] that such a brane world model provides a natural setting for the old idea of Volkov and Akulov to view the neutrino as a Goldstino. Indeed in the simplest example of an M-fivebrane wrapped on a Riemann surface the Fermions appearing in the low energy effective action are Goldstinos. However their low energy dynamics is indistinguishable from that of a theory where they have non-derivative couplings, namely $N = 2$ Yang-Mills. Furthermore M-theory/type IIA duality strongly suggests that in the full M-fivebrane dynamics the Goldstinos must have have non-derivative interactions with the electrically charged states. In other words we see that a soliton/Goldstino non-derivative interaction must exist. Given our discussion in section three we are now in a position to see how this is possible.

In fact the Skryme model discussed above is quite analogous to the situation with the effective theory of the wrapped M-fivebrane. The Skryme model was constructed as a description of the low energy physics of the strong interactions. The pions arise as the Goldstone modes for an $SU_L(2) \times SU_R(2)$ symmetry broken to $SU_D(2)$. Skyrme's insight was to suppose that nucleons could also appear in the low energy description as solitons. Of course, we now know that the fundamental theory of strong interaction is QCD and presumably an action similar to the Skyrme model is derivable from it in a suitable low energy limit.

Thus QCD plays an analogous role to M-theory and the Skyrme model is the counter part of the effective theory of the wrapped M-fivebrane. The fundamental fields on the M-fivebrane arise as Goldstone modes, just as the pions do in the Skyrme model. In addition the nucleons are realised as solitons in the Skryme model and in the effective theory of the wrapped fivebrane all charged states must arise a solitons. Thus the process we wish to study is the scattering of the solitons and the fundamental particles or Goldstone modes in each theory. That is the scattering of the nucleons with pions in the Skyrme model and neutrinos with electrons and $W^{\pm}$'s in the effective theory of the wrapped fivebrane.

These two models have another point in common. In the low energy limit, the lowest order terms for an M-fivebrane wrapped on a Riemann surface are known to be given by the Seiberg-Witten effective action [11]. However, the Seiberg-Witten effective action does not support a finite energy soliton configuration that represents either a monopole or $W^{\pm}$ [28], as one expects from Derricks theorem. However it can be argued that suitable corrections to the Seiberg-Witten dynamics do arise from the M-fivebrane that will support such soliton solutions [28]. Similarly in the Skyrme model the higher derivative term must be added in order to enable the existence of the solitonic nucleons.

However there is an important difference between the two models. Namely the soliton solutions on the M-fivebrane that describe charged states behave as $1/r$ at infinity [28] and not $1/r^2$ as the Skyrmion does. Therefore, according to our argument in section three, there will be non-derivative scattering of the solitons with the Goldstone particles. Although we only considered bosonic internal symmetries in section three we expect an analogous result will be apply for supersymmetries and their corresponding Goldstinos. In these cases one would have to repreat this analysis including spinorial and Fermionic charges. Indeed, as we mentioned in the introduction, in the example of two intersecting M-fivebranes with four-dimensional $N = 2$ supersymmetry, duality implies that these non-derivative interactions must exist on the M-fivebrane since they exist in the gauge theory of the D-fourbranes in type IIA string theory. In this sense the argument in section three provides a test of M-theory/type IIA duality and the relation of the M-fivebrane to quantum gauge theory.

Returning to realistic scenarios the four-dimensional M-fivebrane vacuum itself breaks rigid supersymmetry by wrapping over a non-supersymmetric two-cycle of $\mathcal{M}$, i.e. the four-dimensional vacuum of the M-fivebrane spontaneously breaks rigid supersymmetry. This leads to massless and chargeless Goldstinos in the four-dimensional worldvolume. In addition any electrically charged states arise as soliton solutions to the M-fivebrane worldvolume theory and these necessarily also break the supersymmetry of the M-fivebrane equations of motion. Hence it follows that, in the effective theory of the wrapped fivebrane, the spin 1/2 Goldstinos and the charged particles possess non-derivative interactions and as such it is compatible to identify the Goldstinos with the neutrinos. This is in contrast to the situation considered in [1] where the electron and Goldstinos were both fundamental particles in the theory and their interaction can only involve derivative couplings. Thus when constructing theories of the standard model using wrapped branes one can identify the Goldstinos with the observed neutrinos and not be in contradiction with the low energy theorems.

5. Conclusions

In this paper we have analysed the interactions of Goldstone particles with soliton states and showed that the scattering amplitudes need not vanish in the limit of zero momentum. This allowed us to the resolve a conflict with the duality between type IIA string theory and M-theory, in a sense forming a test of this duality. This discussion also helps elucidate the origin of the minimal coupling of soliton states to the worldvolume gauge field. We also discussed a phenomenological scenario in which neutrinos arise naturally as Goldstino particles for broken supersymmetry but still have non-derivative couplings to the charged fields. It seems reasonable to us that this situation will occur rather generically in other brane world phenomenological models.

It is important to note that there are other problems that arise when the neutrino is identified with a Goldstone mode of broken supersymmetry (e.g. see [35]). For example since the neutrino carries lepton number then presumably so must the corresponding broken supersymmetry generator. Therefore if the supersymmetry is spontaneously broken in the usual sense one expects that there will be massive states which carry both baryon number and lepton number. Such states would then cause problems for phenomenology. However it is not clear that the standard treatment of this issue applies in the case of branes that we have advocated here. For example if some of the supersymmetries are non-linearly realised then there simply may not be any of the corresponding superpartners on the brane. Indeed this is the case for D-branes where the broken sixteen supersymmetries are non-linearly realised in the Dirac-Born-Infeld action and there are no corresponding (massive) superpartners on the worldvolume. Another possiblity is that lepton number is an accidental symmetry, which perhaps arises as a remenant of a discrete symmetry in the supersymmetric action of an unwrapped M-fivebrane.

Another problem that arises in the old four-dimensional $N = 1$ models where the neutrino is identified as a Goldstino mode is that only one such particle, and not three, should appear. However, within the framework discussed here it is possible that several neutrinos are simply related to the breaking of additional supersymmetries.

In closing we note that there is now strong experimental evidence that not all the neutrinos are massless. However, their observed masses are small compared to the other scales in a brane world model and may be neglected to a good approximation. Nevertheless these non-zero masses must be explained at least in principle and one might suppose this was due to a small breaking of supersymmetry in the bulk. Indeed such a small breaking in the bulk spacetime would lead to a non-zero value of the cosmological constant for which there is also recent (although at this stage less reliable) experimental evidence. It would be interesting to see if these two mass scales could be related to each other. Although these scales are still very different such a relation would presumably also involve the compactification scale of the extra dimensions. In other words one can ask the question as to whether or not the small, but non-zero, cosmological constant is due to a small breaking of bulk supersymmetry which in turn gives rise to a mass for any would-be Goldstino neutrinos?

6. Acknowledgements

We would like to thank A. Uranga for discussions and N. Manton for helpful communication on the Skyrme model. We would also like to thank the Theory Division at CERN for its hospitality where this work was initiated. This work was supported in part by the two EU networks entitled "On Integrability, Nonperturbative effects, and Symmetry in Quantum Field Theory" (FMRX-CT96-0012) and "Superstrings" (HPRN-CT-2000-00122). It was also supported by the PPARC special grant PPA/G/S/1998/0061 and N.D.L. is support by a PPARC fellowship.

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